

Supporting Information for Unsupervised fringe pattern analysis based on regularized phase tracking

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ABSTRACT

This document provides supplementary information for “Unsupervised fringe pattern analysis based on regularized phase tracking”.

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Supporting Information Section S1. Comparative experiments across multiple representative single-frame fringe pattern analysis methods

To further validate the advantage of the proposed RPT-ulFPA framework for single-frame fringe analysis, comparative experiments were conducted on the same eye-shaped plaster model using multiple representative single-shot fringe pattern analysis methods. The 12-step phase-shifting profilometry (PSP) results with three-frequency temporal phase unwrapping were used as the ground truth.

Fig. S1 provides comparative 3D reconstruction results of several representative single-frame phase retrieval methods, including Fourier transform profilometry (FTP), windowed Fourier transform (WFT)¹, wavelet transform (WT)², phase retrieval using VID^{3,4} and HST⁵, and classical regularized phase tracking (RPT)⁶. The results show that WFT slightly improves the phase accuracy over FTP due to its localized frequency analysis. However, WT is sensitive to local fringe frequency variations, especially in dense fringes and steep curvature regions, where mismatched wavelet scales make high-frequency components difficult to extract accurately, resulting in unstable local phase estimation and degraded 3D details. For phase retrieval using VID and HST, residual background and modulation errors remain in the extracted orthogonal fringes, producing noticeable ripple-like artifacts in the complex local structures. In the phase retrieval based on the classic RPT, VID and HST are also used for fringe preprocessing, and the corresponding results are shown in Fig. S1(e). Although classical RPT offers a physically interpretable local optimization framework, its reliance on a local phase-plane model and pixel-wise iterative optimization makes it highly sensitive to window size and local frequency estimation, resulting in increased phase errors and reduced completeness near object boundaries. In contrast, the proposed RPT-ulFPA achieves more complete 3D reconstruction results with the highest phase accuracy, as shown in Fig. S1 and Table S1.

The quantitative results are summarized in Table S1. Compared with all representative traditional single-shot fringe pattern analysis methods, the proposed RPT-ulFPA reduces the phase-error MSE by beyond 50%, respectively. These results indicate that the performance improvement does originate from the joint effect of phase-related fringe preprocessing, RPT-inspired differentiable physical constraints, and the two-step refinement strategy.

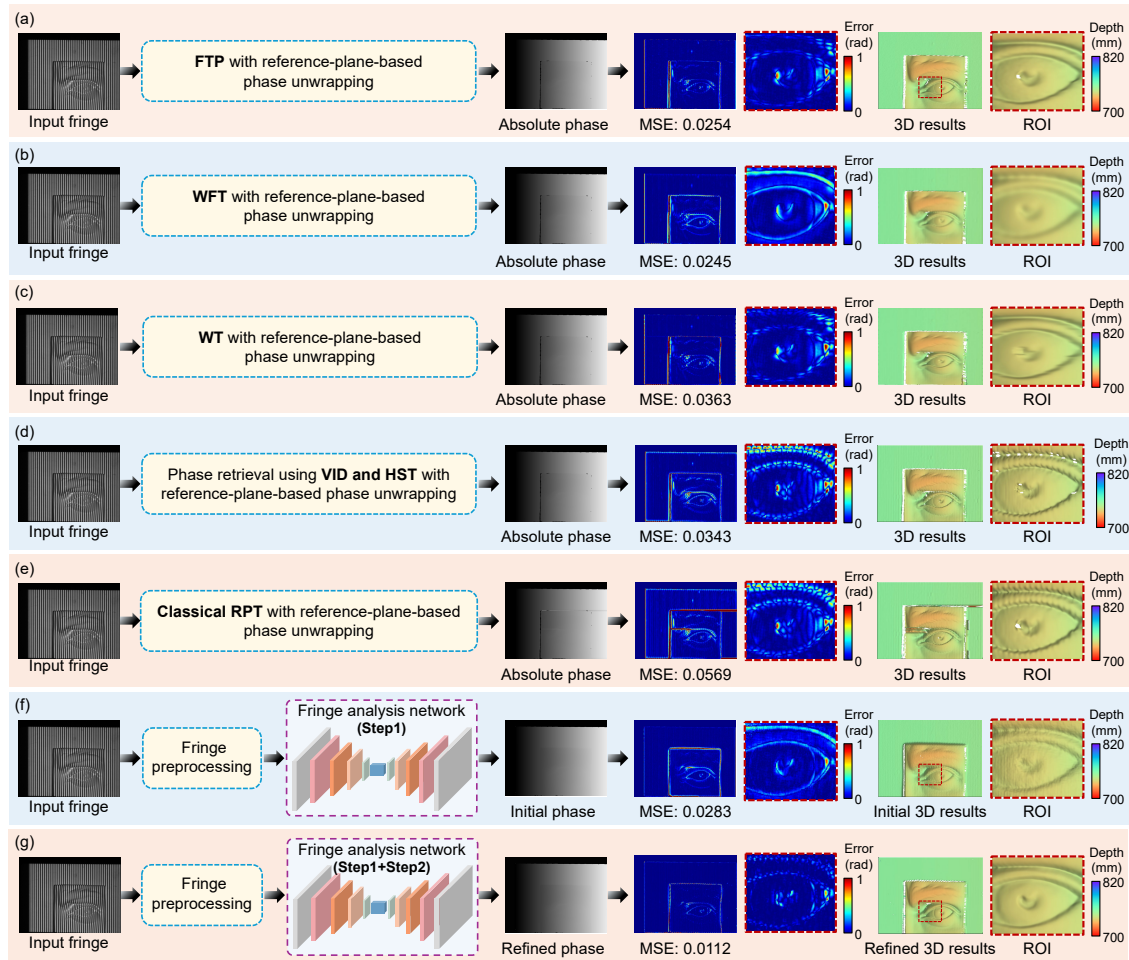


Figure S1. Comparative 3D reconstruction results for different single-shot fringe pattern analysis methods. (a)-(g) The phase retrieval process, the absolute phases, the phase errors with the corresponding magnified views, 3D reconstruction results with the corresponding magnified views using FTP, WFT, WT, phase retrieval using VID and HST, classical RPT with reference-plane-based phase unwrapping, and two refined steps in RPT-ulFPA.

Table S1. Quantitative comparison with representative single-frame phase retrieval methods.

Method	MSE of phase errors (rad)
FTP	0.0254
WFT	0.0245
WT	0.0363
Phase retrieval using VID and HST	0.0343
Classical RPT	0.0569
RPT-ulFPA (Step1)	0.0283
RPT-ulFPA (Step1+Step2)	0.0112

Supporting Information Section S2. Ablation studies for the unsupervised fringe analysis networks

Ablation studies for the unsupervised fringe analysis network were performed to evaluate the individual contributions of the two-step phase optimization strategy and the RPT-inspired loss terms that combine the photometric consistency and the phase regularization as shown in Fig.S2. Four variants were tested on the same eye-shaped plaster model: (a) Step1 of RPT-ulFPA with only the phase smoothness loss ($Loss_{phase}$); (b) Step1+Step2 of RPT-ulFPA with only $Loss_{phase}$; (c) Step1 of RPT-ulFPA with both the photometric consistency loss ($Loss_{irrad}$) and $Loss_{phase}$; and (d) the full model using Step1+Step2 of RPT-ulFPA with $Loss_{irrad}$ and $Loss_{phase}$. The 12-step PSP result was used as the ground truth for calculating the absolute phase error.

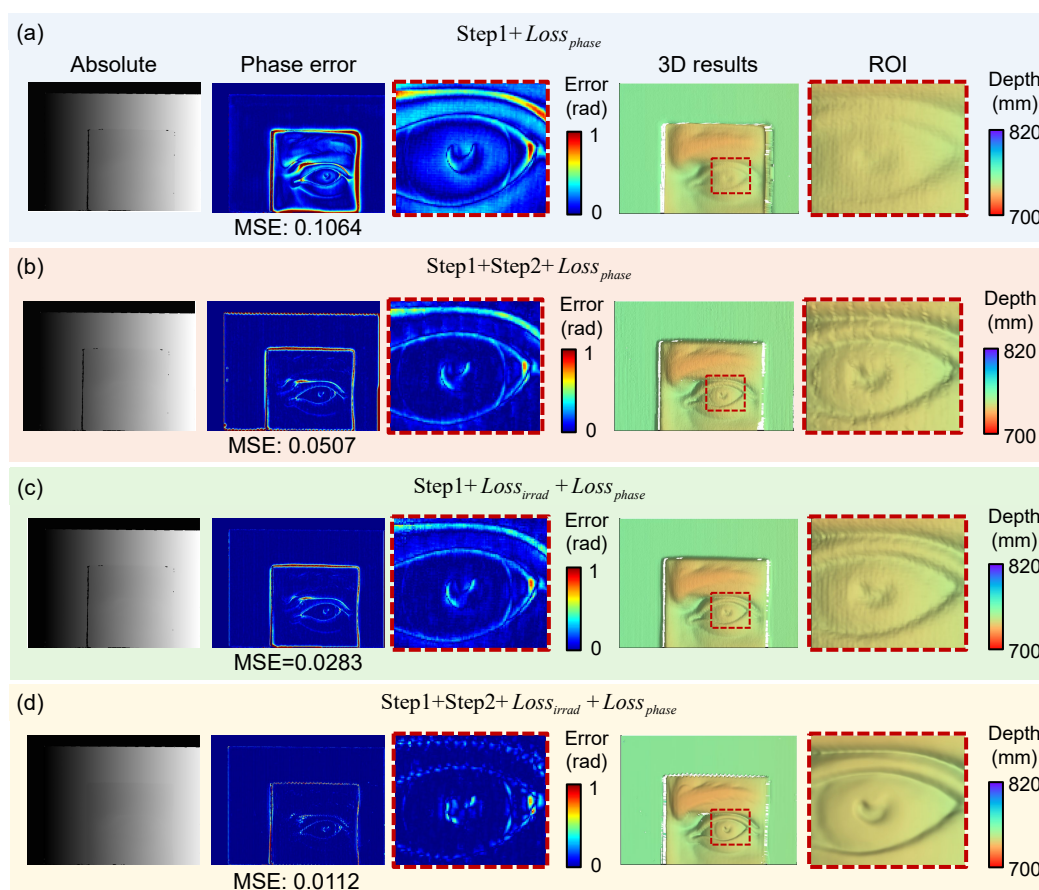


Figure S2. Ablation studies for the unsupervised fringe analysis network. (a)-(d) the absolute phases, the phase errors with the corresponding magnified views, 3D reconstruction with the corresponding magnified views using the four variants Step1+ $Loss_{phase}$, Step1+Step2+ $Loss_{phase}$, Step1+ $Loss_{irrad}$ + $Loss_{phase}$, and Step1+Step2+ $Loss_{irrad}$ + $Loss_{phase}$.

The ablation results in Fig. S2 substantiate two key contributions and Table S2 summarizes the quantitative results. Specifically, as shown in Fig.S2 (a), using only the phase smoothness loss in Step1

produces an over-smoothed and insufficiently constrained solution, leading to the largest phase-error MSE of 0.1064. Introducing the second refinement step under the same $Loss_{phase}$ setting reduces the MSE to 0.0507 from the comparisons of Fig.S2 (a) and (b), showing that the two-step strategy helps improve phase consistency even without the irradiance fidelity term. When the photometric consistency loss is added to Step1 as shown in Fig.S2 (c), the MSE is reduced to 0.0283, confirming that the irradiance constraint is essential for preventing ambiguous smooth phase solutions. Therefore, the pairwise comparisons of (a) with (b) and (c) with (d) confirm that the two-step refinement strategy yields a significant reduction in the phase error. Notably, our complete model yields 3D reconstruction results with the highest phase accuracy, demonstrating that the performance improvement arises from the synergistic integration of the two-step optimization strategy and the RPT-inspired loss terms.

Table S2. Ablation results of different optimization strategies and loss terms.

Model/loss	Step1	Step1+Step2	$Loss_{phase}$	$Loss_{irrad} +$ $Loss_{phase}$	MSE of phase errors (rad)
Step1+ $Loss_{phase}$	✓		✓		0.1064
Step1+Step2+ $Loss_{phase}$		✓	✓		0.0507
Step1+ $Loss_{irrad}$ + $Loss_{phase}$	✓			✓	0.0283
Step1+Step2+ $Loss_{irrad}$ + $Loss_{phase}$		✓		✓	0.0112

Supporting Information Section S3. Network architecture

The proposed RPT-ulFPA adopts a standard U-Net architecture for unsupervised phase estimation. In the current implementation, the network takes a single-channel preprocessed fringe component (I_{cos}) as input and outputs a single-channel phase map. The corresponding orthogonal fringe component is not directly fed into the network, but is used together with the input component to construct the photometric consistency loss. The U-Net consists of a symmetric encoder-decoder structure with skip connections. Each encoder block contains two convolutional layers followed by nonlinear activation, and the spatial resolution is reduced by a factor of two after each downsampling operation. The decoder progressively restores the spatial resolution through upsampling operations and concatenates the corresponding encoder features through skip connections. Finally, a (3×3) convolution maps the decoded feature map to a single-channel phase output. In the second refinement step of the proposed RPT-ulFPA method, a modified sigmoid activation is used to constrain the estimated wrapped phase into the range $[0, 2\pi)$.

For the 640×480 fringe images used in this work, the network architecture can be summarized as shown in Table S3. The exact number of feature channels should be adjusted according to the implemented U-Net configuration.

Table S3. U-Net architecture used in RPT-ulFPA.

Stage	Feature size	Channels	Operation
Input	640×480	1	Preprocessed fringe input I_{cos}
Encoder 1	640×480	32	Two convolutional layers and downsampling
Encoder 2	320×240	64	Two convolutional layers and downsampling
Encoder 3	160×120	128	Two convolutional layers and downsampling
Encoder 4	80×60	256	Two convolutional layers and downsampling
Bottleneck	40×30	512	Two convolutional layers
Decoder 4	80×60	256	Upsampling and skip connection
Decoder 3	160×120	128	Upsampling and skip connection
Decoder 2	320×240	64	Upsampling and skip connection
Decoder 1	640×480	32	Upsampling and skip connection
Output	640×480	1	3×3 convolution for phase estimation

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